Times Tables- Progression and Guidance

Definitions

- Multiplication is a short cut way for adding equal groups, not just repeated addition.
- Knowing a times table is being able to retrieve a fact quickly (automaticity) as well as having conceptual understanding (can't check this for every fact)



a) To develop number sense:

- Pupils can skip counting in tens forward and backwards (to scaffold counting in 1s)
- Pupils can count in 1s to 100, forwards and backwards
- Pupils can fluently say one more one less than any number up to 100*
- Pupils can link counting in 1s to addition and subtraction of 1*
- Pupils can represent adding or subtracting one on a number line and in an equation*

b) To develop understanding of skip counting in steps >1 and the structure of multiplication

- Pupils can unitise
- Pupils can identify equal and unequal groups
- Pupils can make equal groups and describing 3 key elements (e.g. there are 5 groups, there are three in each group, there are 15 altogether)
- Pupils understand doubles as the addition of two equal groups*
- Pupils know their double facts to 20 (e.g. from double 1 to double 10)*
- Pupils begin to understand the early relationship between repeated addition and the times sign

*Doubles and near doubles are Stage 3 Book 6 of our Systematic Number Fact Teaching through Number Sense.

The work on identifying and describing equal groupings is essential if pupils are to understand what a multiplication fact represents.

By the time pupils complete this pre-requisite work, they will already be able to skip count in 10s and it will be an easy step to link the rapid recall of double facts and rapid recall of 2x table facts.



^{*}One more one less is Stage 3 Book 1 of our Systematic Number Fact Teaching through Number Sense.

What do we mean by "fluency in times tables facts?"

This policy seeks to prepare pupils to be "fluent" in times tables facts so that:

1)	Pupils reach "autom	aticity" on mul	tiplication facts ι	up to 12x12	where the	missing
	number is in position	າ three.				

For example: 1	n1 x n2 =
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For the purpose of this policy, we understand "automaticity" to mean instant or near instant recall of answers to multiplication questions. Facts known to automaticity are the "known" facts.

- 2) Pupils are rapidly able to use "known facts" to find related division facts; we are clear that since the soundbites to division facts are not taught, recall will not be instant, but we aim to get pupils to the point where they can very quickly/almost instantly make links between known multiplication facts and related division facts up to 12 x 12
- 3) Pupils are able to use "known" facts to derive missing numbers in multiplication questions where the missing number is not is position 3, but in positions 1 or 2.
- 4) Pupils have an excellent grasp of what multiplication and division facts represent. This means that for any multiplication and division equation, pupils can verbalise two models:

For multiplication, a child could say that 3x2 is

- three groups with two in each group, OR
- three, two times (in other words two groups of three)

For division, a child could say that 10÷5= 2 is

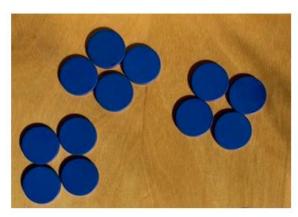
- 10 split up into 5 groups, with two in each group
- 10 split up into groups of 5, which would result in two groups

We recognise the critical importance of children being able to interpret a multiplication or division equation in full sentences; this demonstrates they have grasped the basic structures of multiplication and division and this will be hugely important to their ability to tackle more complex problems involving multiplication and division.

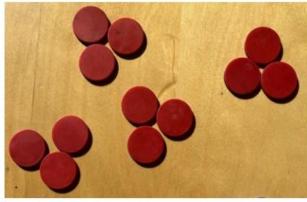
In our early teaching sequences on multiplication and division, we will regularly use manipulatives to help children see and talk about equal groups.

Initially we will NOT use arrays which do not shown groupings unambiguously. This will come later in later teaching sequences. Instead, we will show random clusters such as:

Three groups of four



Four groups of three



Only when children can fluently describe and make groupings in the form above will it be appropriate to introduce arrays to show that an array of 4 x 3 can be interpreted horizontally as 3 groups of four and vertically as four groups of three.

In the early stages of teaching concept, the focus will be on children being able to identify **the three key elements in any multiplication or division scenario**:

- (i) the number of groups/parts
- (ii) the number in each group/ the size of the part
- (iii) the total number / the size of the whole

We will insist on children using full sentences to describe multiplication and division scenarios until they are fluent at this. Only then will multiplication and division equations be used regularly as maths shorthand in lessons which are focussing on developing concept; in lessons which are focussed on developing rote recall of facts, teachers and children will consistently use the agreed fact soundbite. The link between the full verbalisation of a multiplication and division sentence and a fact soundbite should be made regularly so that children make the link between facts known to automaticity and the work they are doing on concept.

Multiplication Progression

By the end of KS1, we want all children to be able to skip count forwards and backwards in steps of 1, 2, 5, 10 to 100. This will hugely strengthen their number sense between 0-100 and will set them up to succeed in KS2 in developing number sense beyond 100. The focus during skip counting in KS1 is not on multiplication and division structures.

We want children to have rapid recall of 2x table facts and 10x table facts by the end of KS1, but the mechanism for developing "automaticity" in these facts should in the first instance be developed through knowledge of doubling facts to 20 for 2x table and strong conceptual understanding of place value for 10x table. In addition, the rote recall techniques described below can also be used but should be done after the work on doubles and Place Value is complete.

The focus in KS1 is not on developing rote recall of facts per se, but on securing a strong sense of the underlying structures of multiplication and division, and the links between them. By the end of KS1 we want children to be fluent in describing the links between a product and its factors in terms of two models of multiplication and two models of division. (See Dottie and Joel video, Card 14).

In KS2, there is a dual focus on deepening conceptual understanding and the ability to reason around multiplication and division structures, and also the acquisition of key facts to automaticity.

Children should be helped to see the acquisition of multiplication facts to automaticity as an opportunity to develop their understanding of how we learn, and to improve their own learning techniques. In other words, learning times table facts can be used positively as an exercise in develop children's metacognition.

Year 1

- Counting accurately in 1's forward and backward to 100
- One more one less
- Counting in tens (first experience of skip counting)
- Doubling and halving (overlap with 2x table)
- Count in 2's, 5's and 10's

Year 2

- Counting accurately in 1's forward and backward to 100
- One more one less
- Counting in tens (first experience of skip counting)
- Doubling and halving (overlap with 2x table)
- Recall the 1x, and corresponding division facts
- Recall the 2, 5 and 10 multiplication tables, and corresponding division facts
- Recall the 0x table

Year 3

- Secure and maintain fluency in addition and subtraction within and across 10 through continued practice
- Recall the 4 and 8 times and corresponding division facts (Building on 2 times table taught in Y2)
- Recall the 3,6,9 multiplication tables, and corresponding division facts

Year 4

- Secure and maintain fluency in addition and subtraction within and across 10 through continued practice
- Recall the 7 multiplication table and corresponding division facts (introduced in Y3 PDno)
- Recall the 11 and 12 multiplication tables, and corresponding division facts

Y5/Y6

- Secure and maintain fluency in all multiplication tables, and corresponding division facts, through continued practice (all covered Y2-4)
- Prime numbers
- All possible factors
- Related facts
- Related facts decimals
- Apply times table facts to problems

NCETM PD Spine 2: Multiplication and Division

The multiplication facts for each times table are taught in depth in the following PD materials. Focus during main maths lessons will be conceptual understanding of the multiplication facts and links to prerequisite times tables.

Year 1

• 2.1 Counting, unitising and coins

Year 2

- 2.3 Times tables: groups of 2 and commutativity (part 1)
- 2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1
- 2.5 Commutativity (part 2), doubling and halving

Year 3

- 2.7 Times tables: 2, 4 and 8, and the relationship between them
- 2.8 Times tables: 3, 6 and 9, and the relationship between them
- 2.9 Times tables: 7 and patterns within/across times tables

Year 4

• 2.11 Times tables: 11 and 12

Steps to teaching (rote recall)

Times table rehearsal sessions Components

- Regular rehearsal practice to develop fluency of your year groups times tables (3-5 mins, 3-5 times per week)
- This will consolidate work in the PD lessons where the times table is explored further and in more depth.

Times table introduction

Focus on **one times table each half term**- with opportunities built in to practice those learnt previously. Children will learn the times table in this order, so that they can make connections more easily.

Year 2					Year 3					Year 4			
	х1	x2	х5	x10	х0	х4	х8	х3	х6	х9	х7	x11	x12

YEAR	AUT 1	AUT 2	SPR 1	SPR 2	SUM 1	SUM 2				
1	Experience of counting in 1s and skip counting in 2s, 5s, 10s									
2	Consolidation of Y1	x1	2x (1x)	5x	10x (5x)	0x				
3	Consolidation of Y2	4x (2x)	8x (4x)	3x	6x (3x)	9x				
4	4 7x 11x		12x	Squares	Consolidation	Test- June				

Presentation of times table

- x 6 is the 6 times table.
- So 4 x 6 is 4 sixes.
- Concrete modelling will be used to support this, see PD materials and examples in the appendix.

Soundbites

Teachers use short form to ask questions and pupils answer in full form in order to hear the whole soundbite. For example:

Sound bite for Multiplication:

- 4 x 6 =

o Teacher: 4 sixes?

o Children: 4 sixes are 24

Soundbite same for Division

- 63 ÷ 7 = ____

Teacher say _um_ sevens are 63

o Children: 9 sevens are 6

Rote recall:

- practicing rote recall is to do with auditory memory

- consistency in the sound bite heard

- short and rhythmical

- support auditory memory with visual clues- Funkey maths cards

Rehearsal

- New facts can be given for pupils to refer to whilst practicing

- 3 min blast

- Need to hear the soundbite over and over

is	2 Is are 2	3 Is are 3	4 ls are 4	5 ls are 5	6 ls are 6	7 Is are 7	8 Is are 8	9 ls are 9	10 ls are 10
1 2 is 2	2 2 s are 4	3 2 s are 6	4 2s are 8	5 2 s are 10	6 2 s are 12	7 2s are 14	8 2 s are 16	9 2s are 18	10 2 s are 20
1 3 is 3	2 3 s are 6	3 3 s are 9	4 3s are 12	5 3 s are 15	6 3 s are 18	7 3 s are 21	8 3s are 24	9 3s are 27	10 3 s are 30
 	2 4 s are 8	3 4 s are 12	4 4 s are 16	5 4s are 20	6 4s are 24	7 4s are 28	8 4s are 32	9 4s are 36	10 4 s are 40
1 5 is 5	2 5s are 10	3 5 s are 15	4 5s are 20	5 5s are 25	6 5s are 30	7 5s are 35	8 5s are 40	9 5s are 45	10 5s are 50
1 6 is 6	2 6 s are 12	3 6 s are 18	4 6s are 24	5 6 s are 30	6 6 s are 36	7 6 s are 42	8 6 s are 48	9 6 s are 54	10 6 s are 60
1 7 is 7	2 7s are 14	3 7 s are 21	4 7s are 24	5 7 s are 30	6 7s are 42	7 7s are 49	8 7 s are 56	9 7s are 63	10 7 s are 70
1 8 is 8	2 8 s are 16	3 8 s are 24	4 8 s are 32	5 8s are 40	6 8s are 48	7 8 s are 56	8 8 s are 64	9 8s are 72	10 8 s are 80
19 is 9	2 9 s are 18	3 9 s are 27	4 9s are 36	5 9s are 45	6 9s are 54	7 9s are 63	8 9s are 72	9 9 s are 81	10 9 s are 90
1 10 is 10	2 10 s are 18	3 10s are 27	4 10s are 36	5 10s are 45	6 10s are 54	7 10s are 63	8 10s are 72	9 10s are 90	10 10 s are 100

Steps to teaching (conceptual)

- When introducing a new times table, build it with the children starting with **known facts** the facts they already know and have met before from the times tables in the previous year group.
- Introduce a new times table by first making clear <u>conceptual links</u> to the real world- half termly display of "what comes in..." the times table you are focusing on. (see Appendix)
 (Build on the teaching they have already had from the PD materials).
- Use a <u>concrete, pictorial and abstract approach</u> to Introduce a new times table for ALL children, but carefully consider the most powerful representation for exposing the mathematical structure, enabling clear connections to be seen, and deeper understanding to be made. Use arrays to model
- Regular <u>retrieval practice</u> to develop fluency building around facts known facts (3-5 mins, 3-5 times per week)
 - Include conceptual support for all children
 - Full verbal patterning (soundbites)
 - Bank of high quality activities for retrieval
 - First in order then out of order
 - Saying it backwards
 - Build in tests, but not as main activity
- Take time to **explore important patterns** within each new times table
 - patterns of doubles within a times table (there are 6 doubles in each x table)
 - spotting patterns of odd and even numbers within a times table

Other patterns such as repeating digits, reversing digits, sum of digits, divisibility tests can be looked at but aren't as important as the ones above.

- Develop mastery through the use of <u>variation</u> (not variety) and <u>intelligent practice</u>.
- Build up to related facts (I know... so)
 - Recall $7 \times 4 = 28$
 - Commutative fact $4 \times 7 = 28$ so 7x4=28
 - 70 x 4 = 280
 - $-0.7 \times 4 = 2.8$
 - Related division facts e.g. I know 7x4 = 28 so $28 \div 7 = 4$ and $28 \div 4 = 7$
 - Distributive law (5 x 4) + (2 x 4) = 7 x 4
 - Distributive law (5 x 7) 7 = 7 x 4
 - Doubling and halving (e.g. 7x4 = 14x2, $28 \div 4 = 7$, $28 \div 2 = 14$)

- Associative law: 3 or more factors, doesn't matter how you group the factors, the product remains the same.

2 x **2** x 3 = **4** x 3 = 12

2 x **2 x 3** = 2 x **6** = 12

Appendix: Definitions

This is common language that the children to need to be exposed to and be used with the children. The language will be introduced during the PD multiplication teaching spines and can be embedded and reenforced during Times tables rehearsal sessions.

Skip counting: The method of counting forward by numbers other than 1.

To skip count, we keep adding the same number each time to the previous number.

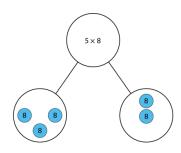
- back up strategy of rote recall
- secures number sense
- useful as + and strategies

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Known facts: Facts that can be rapidly recalled- known to automaticity

Derived facts: Facts that can be worked out from a known fact by using a calculation strategy

Distributive law:



When you multiply one factor by another, you can partition either factor and multiply its parts by the other factor.

Example 1:
$$\underline{\mathbf{5}} \times 8 = \underline{\mathbf{3}} \times 8 + \underline{\mathbf{2}} \times 8$$
 (5 partitioned into 3 and 2)

Example 2:
$$4 \times 12 = 4 \times 10 + 4 \times 2 \text{ (12 partitioned into 10 and 2)}$$

$$48 = 40 + 8$$

40 = 24 + 16

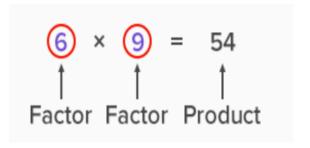
Commutative law: Commutativity is used in Maths equations and describes sums that can be moved around and will still give the same answer. 'Commutative' comes from the word 'commute' which means to move and travel around, so equations that are commutative have numbers that can be moved within the equation.

$$2 \times 7 = 7 \times 2$$

Factor: Factors are positive integers that can be multiplied together to equal a given number.

Factors can have different roles:

- a) They can represent the number of groups
- b) They can represent how many are in each group



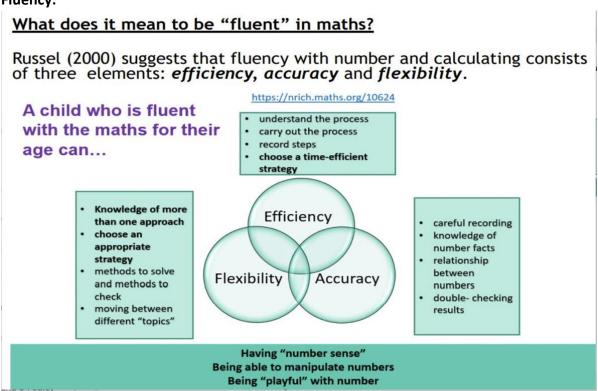
It is important for pupils to be able to see the different roles of the factors.

Product: The result of two factors being multiplied together

E.g. 7 x 3 = 21 (21 is the product of the factors 7 and 3 being multiplied together

Automaticity: "Memorization of basic facts usually refers to committing the result of operations to memory so that thinking is unnecessary...Teaching facts for automaticity in contrast relies on thinking. Answers to facts must be automatic, but thinking about the relationships among the facts is critical. A child can then think of 9x6 as (10x6)-6." Fosnot and Dolk (2001:85) cited in Field, J (2020)

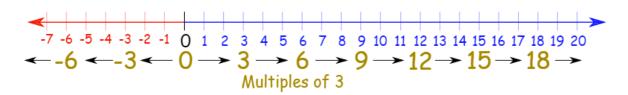
Fluency:



Generalisations: To generalise means to make a statement which holds in all cases, not just in some particular cases. A generalisation is often expressed algebraically.

Multiple: The result of multiplying a number by an integer (not by a fraction). Examples:

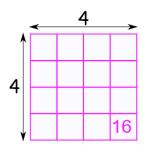
- 12 is a multiple of 3, because $3 \times 4 = 12$
- -6 is a multiple of 3, because $3 \times -2 = -6$
- But 7 is NOT a multiple of 3



Square: The result of an integer (not a fraction) being multiplied by itself. **E.g. 4** x **4** = **16** therefore **16** is a square number.

First 20 square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 361, 400.

Square numbers are also composite numbers.



Prime: Whole numbers greater than 1, that have only two factors – 1 and the number itself.

Prime numbers are divisible only by the number 1 or itself.

Whole numbers that are not prime are composite.

The grid shows all the prime numbers less than 100-they are marked in yellow.

Fun facts

- 1 is neither a prime number or a composite number
- The only even prime number is 2
- No prime number greater than 5 ends in 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Composite: Whole numbers that have more than two factors.

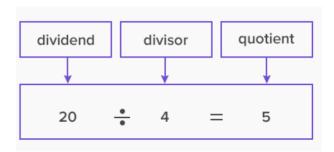
Whole numbers that are not prime are composite numbers, because they are divisible by more than two numbers.

All whole numbers above 1 are either composite or prime. The composite numbers can be found in the grid- they are the numbers **not** highlighted in yellow

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Dividend- the amount or number to be divided. The whole that is to be divided into parts.

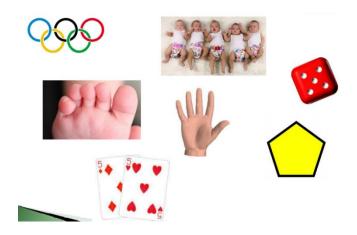
Divisor – a number that divides another number either completely or with a remainder



Quotient- the result when you divide one number by another

Appendix: Ideas for the classroom

Make real world links:



Known Facts:

Build the new timetable around facts the children are confident with and have come across before with commutitive facts.

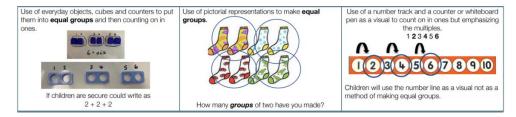
For example x8. They would have learnt their 1,2,4,5 and 10 times tables already. Which facts are left to learn?

Which facts might help us work out the ones that are left?

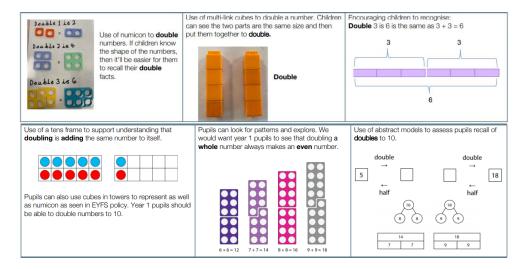
Concrete, Pictorial, abstract:

Be clear which representation you will use and why.

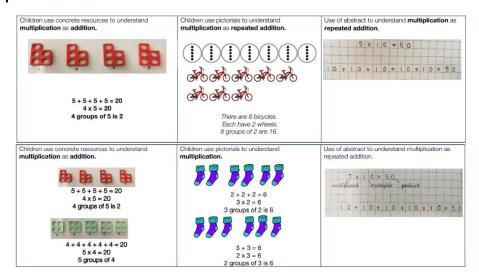
- Equal groups



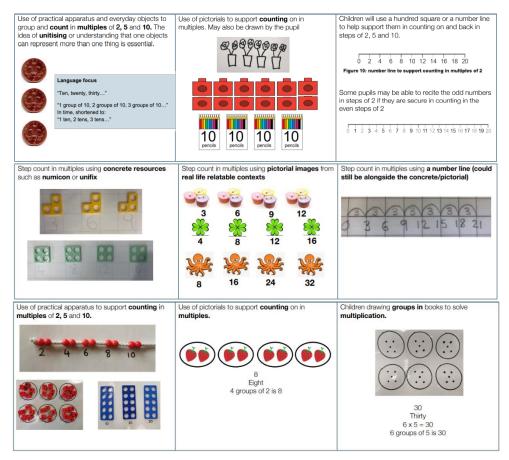
- Doubling



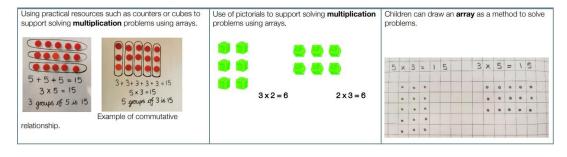
Repeated Addition



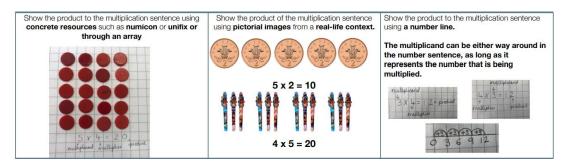
- **Count in multiples.** Using a range of different representations – moving towards a more abstract representation.



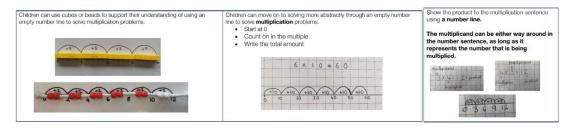
 Arrays for multiplication. Arrays are the most versatile model for modelling the properties of multiplication (repeated addition, commutative, distributive, associative, inverse of division).



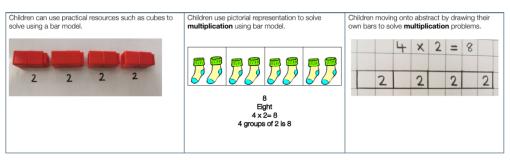
Multiplication equations

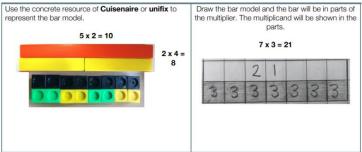


- Number line. This show skips counting.

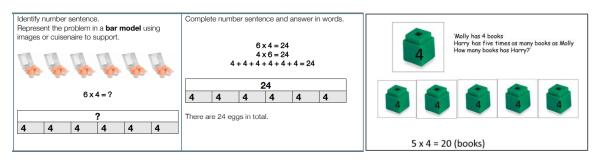


Bar model for representing multiplication.

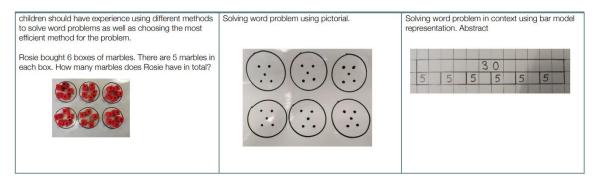




Bar model for representing multiplication problems

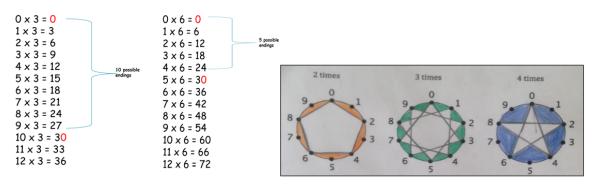


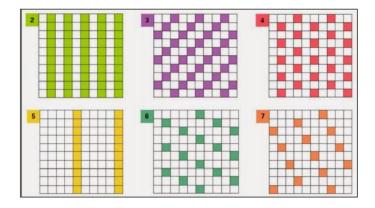
Solving problems in context.



Patterns:

Take time to explore the patterns of each times table as you introduce it to the class. Provide opportunities to deepen knowledge by make generalisations, reasoning, making connections, predicting and explaining.





Intelligent practise:

$$2 \times 3 =$$
 $6 \times 7 =$ $9 \times 8 =$ $2 \times 30 =$ $6 \times 70 =$ $9 \times 80 =$ $2 \times 300 =$ $6 \times 700 =$ $9 \times 800 =$ $20 \times 3 =$ $60 \times 7 =$ $90 \times 8 =$ $200 \times 3 =$ $600 \times 7 =$ $900 \times 8 =$

$$3 \times \square + 2 = 20$$
 $3 \times \square + 2 = 23$
 $3 \times \square + 2 = 26$
 $3 \times \square + 2 = 29$
 $3 \times \square + 2 = 35$

$$4 \times 5 = 10 \square 10$$

 $6 \square 5 = 15 + 15$

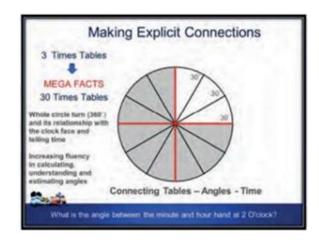
Problem Solving:

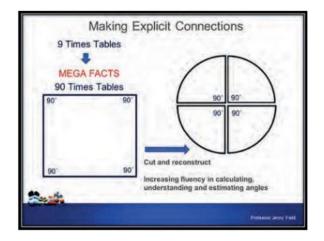
- Always sometimes never
- Models and stories to match an equation.
- What is the same? What is different?
- True or false?

Rehearsal suggestions:

- Funkey maths cards
- counting stick or hoop
- skip counting using manipulatives
- chanting forwards and backwards
- technology based such as TT Rockstars/Number Gym
- quick reaction exercises such as
- Introduce a fact of the day/week
- Display- table focus- facts are added as they are taught, building around what is already known

Mini and mega facts: Connectionist approach (Askew, 1997)





36 facts to take us up to 9 x 9 – Building block facts

Year 3	Year 3	Year 3	Year 4	Year 4	Year 4	Year 4	Year 4
2 x	5 x	3 x	4 x	6 x	7 x	8 x	9 x
2 x 2							
3 x 2	3 x 5	3 x 3					
4 x 2	4 x 5	4 x 3	4 x 4				
5 x 2	5 x 5						
6 x 2	6 x 5	6 x 3	6 x 4	6 x 6			
7 x 2	7 x 5	7 x 3	7 x 4	7 x 6	7 x 7		
8 x 2	8 x 5	8 x 3	8 x 4	8 x 6	8 x 7	8 x 8	
9 x 2	9 x 5	9 x 3	9 x 4	9 x 6	9 x 7	9 x 8	9 x 9
8 facts	7 facts	6 facts	5 facts	4 facts	3 facts	2 facts	1 fact

By end of Y3:

By end of Y4

21 facts learnt

15 facts learnt to complete building blocks

15 facts still to learn 21 more facts for times table check (see below)

- 2x11, 3x11, 4x11, 5x11, 6x11, 7x11, 8x11, 9x11, 10x11, 11x11, 12x11
- 2x12, 3x12, 4x12, 5x12, 6x12, 7x2, 8x12, 9x12, 10x12, 12x12